Deposition of a viscous fluid on the wall of a tube

By G. I. TAYLOR
Cavendish Laboratory, Cambridge

(Received 27 September 1960)

Measurements of the amount of fluid left behind when a viscous liquid is blown from an open-ended tube are described.

Introduction

When air is blown into one end of a tube containing a viscous fluid it forms a round-ended column which travels down the tube forcing some of the liquid out at the far end and leaving a fraction m in the form of a layer covering the wall. This fraction has been measured in some cases by Fairbrother & Stubbs (1935), who were interested in the question whether a bubble of air in a capillary tube through which a fluid is flowing is a true index of the velocity of the flow. If U is the velocity of a bubble and U_m the mean velocity of the fluid ahead of it,

$$m = \frac{U - U_m}{U}. \tag{1}$$

Fairbrother & Stubbs's experiments were limited to the case when m is small, and in that case they found as an empirical relationship, which covered experiments made with fluids in which the velocity U, the surface tension T and the viscosity μ all varied,

$$m = 1{\cdot}0(\mu U/T)^{\frac{1}{2}}. \tag{2}$$

As these authors point out, it is to be expected that m should be a function of $\mu U/T$ since that is the only non-dimensional combination of these three symbols, but that m should be proportional to $(\mu U/T)^{\frac{1}{2}}$ has not been explained, nor has the coincidence that the empirically determined constant is 1·0. Fairbrother & Stubbs point out that there are theoretical and experimental reasons for believing that the thickness of the film left on a plane sheet when it is pulled vertically out of a fluid is proportional to $U^{\frac{1}{2}}$. This, however, is not a proper analogy since the existence of the film is due to a balance of viscous and gravitational forces, whereas when the fluid is blown out of a horizontal tube gravity plays little part in the phenomenon.

The empirical relationship (2) can only be valid when m is small; in fact if $\mu U/T$ is greater than 1 so that the corresponding value of m is greater than 1, (2) is meaningless. In the experiments to be described it was found that (2) is a good approximation in the range $0 < \mu U/T < 0{\cdot}09$. The maximum value of $\mu U/T$ obtained in Fairbrother & Stubbs's measurements was 0·014 so that they were well within the range in which (2) holds. When $\mu U/T$ increases beyond 0·09, m is less than would be predicted by (2). The experiments were extended, using very viscous fluids, to $\mu U/T = 1{\cdot}9$ (i.e. to 135 times Fairbrother & Stubbs's maximum value) when m had reached 0·55. The apparent trend of the curve of experimental points seemed to indicate that m was approaching an asymptotic value at that stage. It seems therefore that if one attempts to blow a very viscous fluid out of a tube more than half may be left behind when the air column breaks through at the far end of the tube.

Experimental details

The method adopted was similar to that used by Fairbrother & Stubbs, but since only one long interface between air and fluid, rather than separate bubbles, was required, the method of introducing the air column needed careful design. The apparatus is shown diagrammatically in figure 1. Since the object of the experiment was to obtain large values of $\mu U/T$ without increasing the speed of flow to such an extent that the stresses due to inertia were comparable with those

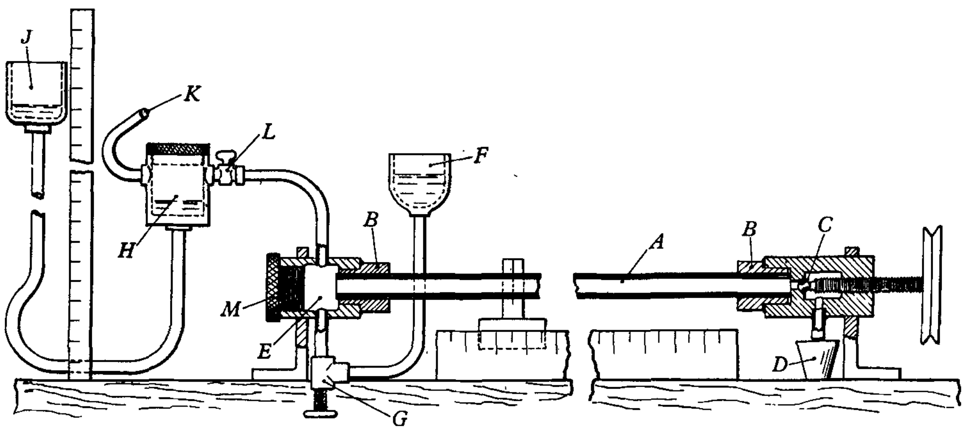

FIGURE 1. Sketch of apparatus.

due to viscosity, most of the experiments were carried out with fluids of large viscosity. Glycerine and strong sucrose solution (golden syrup) diluted with water till its viscosity was 28 poise at 20 °C was used. Glass tubes of accurate bore, known as 'Veridia' tubes, of 2 and 3 mm bore and 4 ft. long were used. The 3 mm tube was slightly conical so that its cross-section varied uniformly through a range of $1\frac{1}{2}$ % over its length. The 2 mm tube had only a quarter of that variation. In figure 1 the tube A has interchangeable brass end pieces B. At the far end these fitted into a fixed ball valve C. The fluid passing the ball passed into a weighed crucible D so that the amount discharged in a measured time could be weighed. At the near end the tube fitted into a chamber E which could be filled with fluid from the container F through the valve G. The air which was forced through the tube was supplied from the Perspex chamber H, and the pressure in H could be raised to 3 atmospheres or more by raising a mercury container J. To maintain the mercury level in H constant the air pressure was raised by means of a subsidiary pump K, while the container J was raised. The pressure vessel could be cut off from the chamber H by turning a tap L.

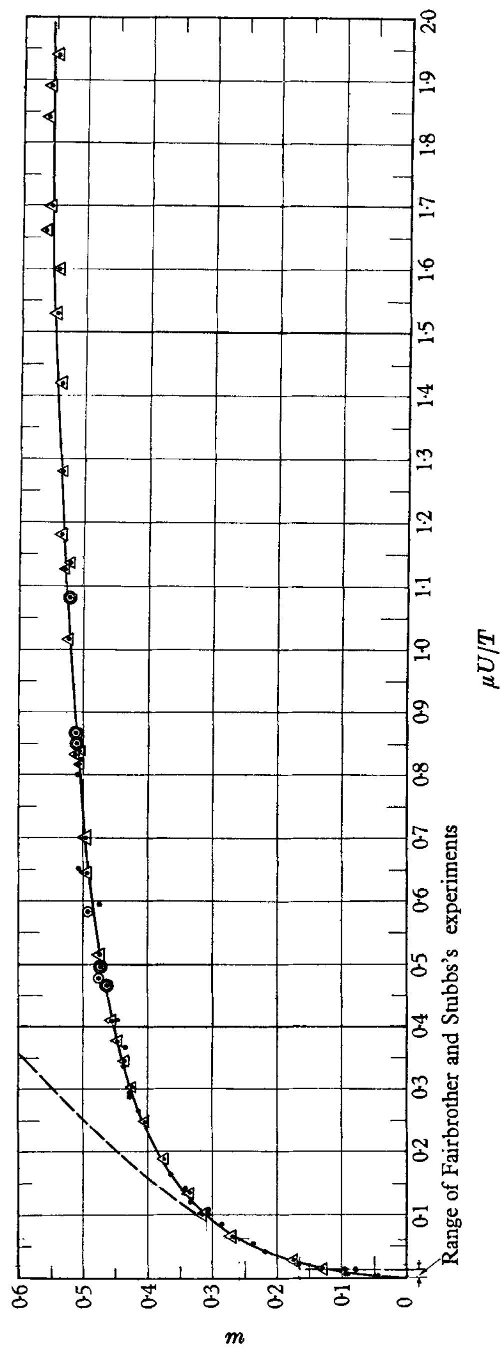

FIGURE 2. Experimental results. •, $\mu = 0{\cdot}79p$ at 20 °C (syrup-water mixture); tube diameters 0·15 and 0·2 cm. △, $\mu = 9{\cdot}3p$ at 20 °C (glycerine); tube diameters 0·2 and 0·3 cm. ◎, $\mu = 28p$ (syrup-water); tube diameter 0·3 cm. ⊙, $\mu = 0{\cdot}3p$ (a lubricating oil); tube diameter 0·15 cm. Broken line: Fairbrother & Stubbs's parabola.

To perform an experiment the tap L was closed and the valves G and C opened. The chamber E and the tube A were then filled, if necessary by applying air pressure to the container F. The valve C was then closed and the crucible D placed in position. By unscrewing a cover M the chamber E was then emptied leaving a meniscus of fluid in the end of the tube. After replacing the cover M the tap L was opened and the air pressure in E raised to about 3 atmospheres.

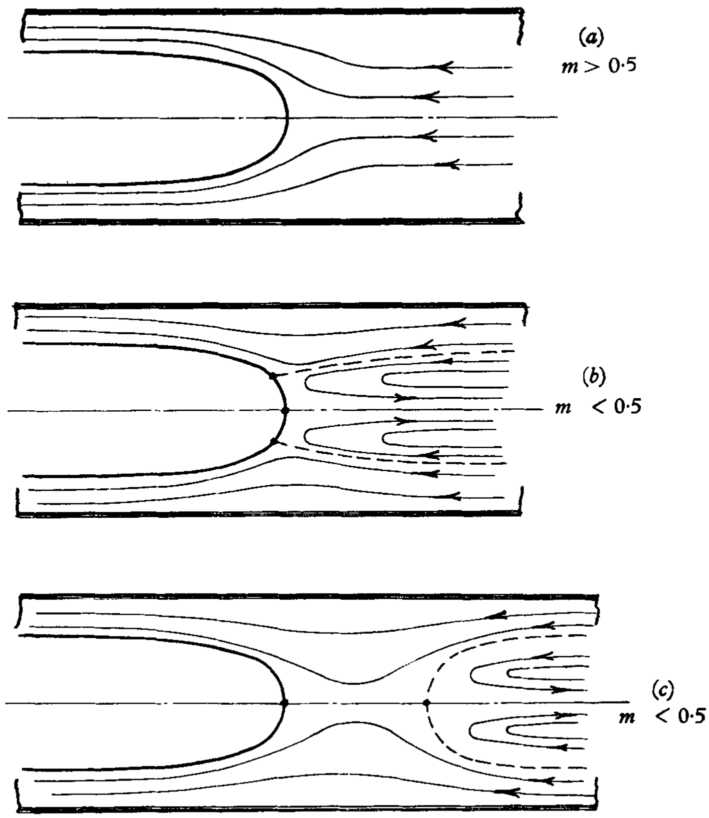

FIGURE 3. Rough sketch of possible streamlines.

The apparatus was then ready for the experiment. The ball valve C was suddenly turned to a previously determined position and at the same moment a stop-watch was started. The meniscus then moved at a nearly uniform speed along the tube. After the meniscus had gone some way along the tube the ball valve C was suddenly closed and simultaneously the stop-watch was stopped. The position of the meniscus was read and the length of the air column in the tube determined. The crucible was weighed and the weight w per unit length of the fluid expelled was measured. The weight per unit length of the fluid in the tube when full was $w_0 = \rho\pi a^2$, where ρ is its density and a the radius of the tube. Evidently

$$1 - m = w/w_0.$$

The viscosity of the fluid used was determined by the capillary-tube method and the surface tensions of glycerine and concentrated solutions of sucrose were taken from physical tables.

Results

The results are shown in figure 2 where m is plotted against $\mu U/T$.

For comparison with the measurements of Fairbrother & Stubbs the parabola $m = (\mu U/T)^{\frac{1}{2}}$ is shown as a broken line, and the range of values of $\mu U/T$ covered by them is also shown. It will be seen that the present experiments are in good agreement with $m = (\mu U/T)^{\frac{1}{2}}$ up to $\mu U/T = 0{\cdot}09$, but that above this value m increases more slowly than $(\mu U/T)^{\frac{1}{2}}$ till at the highest values reached m approaches 0·55. The trend of the curve suggests that m reaches a limiting value above 0·56, when the stresses due to viscosity are much greater than those due to surface tension. It will be noticed that when $m = 0{\cdot}5$ the flow velocity at points far from the meniscus is identical with that of the meniscus, so that if $m > 0{\cdot}5$ the central filament is moving towards the meniscus, whereas if $m < 0{\cdot}5$ the central filament is moving away from it. If the flow is reduced to steady motion by superposing a velocity $-U$ it is only when $m > 0{\cdot}5$ that the flow with only one stagnation point is possible. This is shown by the sketch of figure 3*a*. The two simplest possible types of flow when $m < 0{\cdot}5$ are shown in figures 3*b* and 3*c*. In 3*b* there is one stagnation point on the vertex and a stagnation ring on the meniscus. In figure 3*c* there are two stagnation points on the axis.

REFERENCES

Fairbrother, F. & Stubbs, A. E. 1935 *J. Chem. Soc.* **1**, 527.